Self-organized Boolean game on networks

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A model of a Boolean game with only one free parameter p that denotes the strength of local interaction is proposed wherein each agent acts according to the information obtained from his neighbors in the network, and those in the minority are rewarded. The simulation results indicate that the dynamic of the system is sensitive to network topology, whereby the network of larger degree variance, i.e., the system of greater information heterogeneity, leads to less system profit. The system can self-organize to a stable state and perform better than the random choice game, although only the local information is available to the agents. In addition, in heterogeneity networks, the agents with more information gain more than those with less information for a wide extent of interaction strength p.

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I. INTRODUCTION

Complex adaptive systems composed of agents under mutual influence have attracted considerable interest in recent years. It is not unexpected that the systems with globally shared information can be organized. A basic question in studies of complexity is how large systems with only local information available to the agents may become complex through a self-organized dynamical process [1].

The mutual influence can be properly described as the so-called information network, in which the nodes represent agents and the directed edge from x to y means the agent y can obtain information from x. For simplicity, undirected networks are considered in this paper. In this way, node degree k is proportional to the quantity of information available to the corresponding agent. The two extensively studied information networks of ecosystems are regular [2–4] and random [1,5] networks, both of which have a characterized degree—the mean degree $\langle k\rangle$. For regular networks, all the nodes are of degree $\langle k\rangle$, and for random ones, the degree distribution decays quickly in a Poissonian form when $k > \langle k\rangle$. The existence of a characterized degree means every node has almost the same capacity of information. However, previous empirical studies have revealed that the information networks may be of scale-free property [6–8], in which a giant heterogeneity of information exists. The nodes of larger degree contain much more information than those of less degree, thus, the information heterogeneity can be measured by the degree variance $\langle k^2\rangle$. The question is how the topology affects the system dynamic: will the greater information heterogeneity induce more profit for the system, or less?

Another question of concern in this paper is herd behavior, which has been extensively studied in Behavioral Finance and is usually considered as one factor of the origins of complexity that may enhance the fluctuation and reduce the system profit [9–13]. Here we argue that, to measure the underlying possibility of the occurrence of herd behavior, it is more proper to look at how much the agents' actions are determined by others rather than how much the agents want to be in majority, since in many real-life cases, the agents would like to be in minority but the herd behavior still occurs. We wonder whether agents have different responses under a fixed interaction strength, and whether the varying trends of system profit and individual profit are the same as the increase of interaction strength.

In this paper, a model of a Boolean game with only one free parameter p that denotes the strength of local interaction is proposed, whereby each agent acts according to the information obtained from his neighbors in the network and those in the minority are rewarded. Although the model may be too simple and rough, it offers a starting point aiming at the questions above. We have found that the topology of the information network affects the system dynamic much and that the system can self-organize to a stable state with more profit compared with the random choice game even only the local information is available.

II. MODEL

The Boolean game, first proposed by Kauffman, is set up so that each agent has only one binary choice, such as either buying or selling a stock [14]. The studies of the Boolean game have attracted not only physicists' but also ecologists' and economists' attention since it could explain much empirical data and might contribute to the understanding of the underlying mechanisms of the many-body ecosystems, although the dynamic rule is simple [1,5,15].

Inspired by the idea of the minority game [16], which is a simple but rich model describing a population of selfish individuals fighting for a common resource, we propose the present Boolean game wherein each agent chooses between two opposing actions, simplified as +1 and −1, and the agents in the minority are rewarded. Each winner's score

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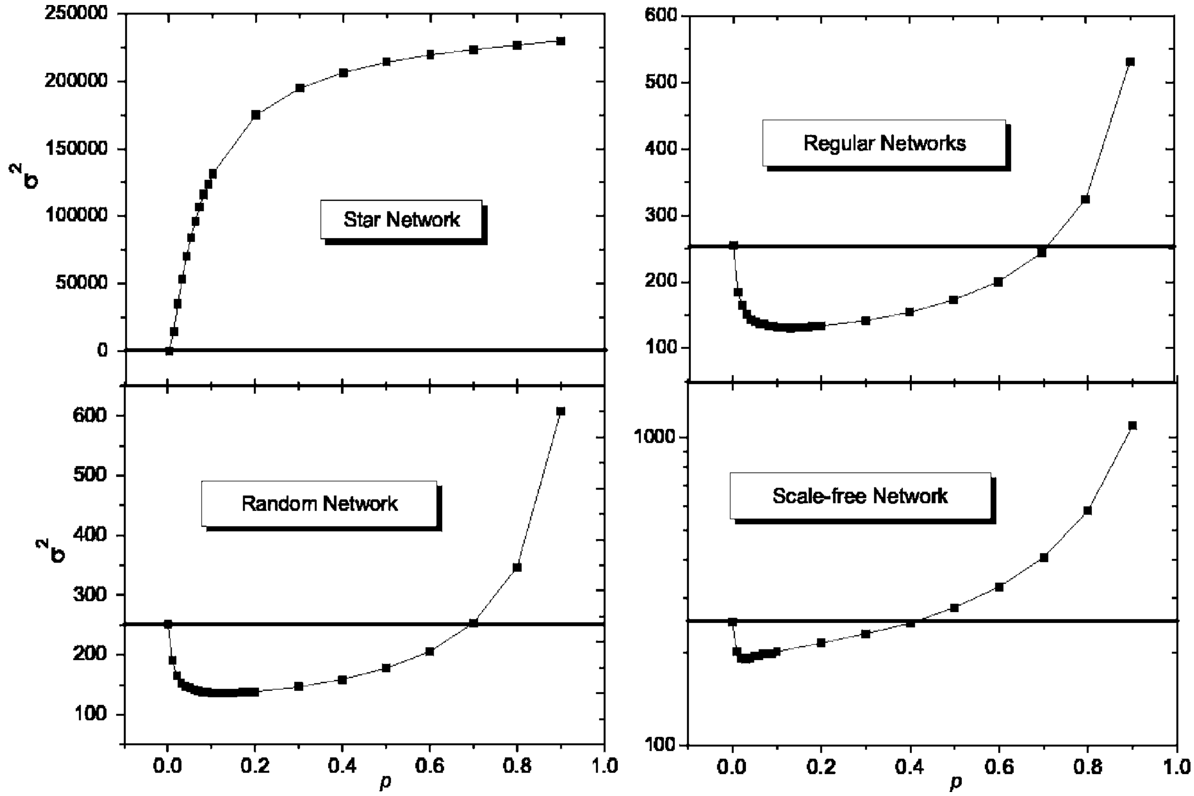

FIG. 1. The variance of the number of agents choosing +1 as a function of interaction strength p. The four plots are the cases of star, regular, random, and scale-free networks. The solid line represents the random choice game, where $\sigma^2=0.25N$. It is clear that the system profit is more than the random choice game when $p\in(0,0.7)$, $p\in(0,0.7)$, and $p\in(0,0.4)$ in regular, random, and scale-free networks, respectively. For any $p\in(0,1)$, σ^2 of the four cases satisfies the relations $\sigma^2_{\text{regular}}<\sigma^2_{\text{random}}<\sigma^2_{\text{scale-free}}<\sigma^2_{\text{star}}$, which means the system profit S satisfies $S_{\text{regular}}>S_{\text{random}}>S_{\text{scale-free}}>S_{\text{star}}$.

increases by 1, thus the system profit equals to the number of winners [17–19]. In our model, at each time step, each agent acts based on his neighbors at probability p, or acts all by himself at probability $1-p$. In the former case, we assume each neighbor has the same force. Since the arbitrary agent x would like to be in the minority, he will choose +1 at the probability $s^x_{-1}/(s^x_{-1}+s^x_{+1})$, or choose −1 at the probability $s^x_{+1}/(s^x_{-1}+s^x_{+1})$, where s^x_{-1} and s^x_{+1} denote the number of x's neighbors choosing −1 and +1 in the last time step, respectively. In the latter case, since there is no information from others, the agent simply inherits his action in the last time step or chooses the opposite action at a small probability m, named the mutation probability. It is worthwhile to emphasize that the agents do not know who are winners in the previous steps since the global information is not available, which is also one of the main differences from the previous studies on the minority game.

The real-life ecosystem often seems a black box to us: the outcome may be observed, but the underlying mechanism is not visible. If we see many agents display the same action, we say herd behavior occurs, although those agents might prefer to be in the minority. From another point of view, if each agent acts all by himself, there is no preferential choice for +1 and −1 so that no herd behavior will occur. Therefore, if herd behavior occurs, the agents' actions must be at least partly based on the information obtained from others. In this paper, the underlying possibility of the occurrence of herd behavior is measured by how much the agents' actions are determined by others, that is to say, by the interaction strength p.

III. SIMULATIONS

In this paper, all the simulation results are the average of 100 realizations, and for each realization, the time length is $T=10^4$ unless a special statement is addressed. The number of agents $N=1001$ and mutation probability $m=0.01$ are fixed. Figure 1 shows the variance $\sigma^2=(1/T)\Sigma_{t=1}^{T}(A_t-N/2)^2$ as a function of p in star, regular, random, and scale-free networks, where A_t is the number of agents who choose +1 at time step t. Clearly, the smaller σ^2 corresponds to more system profit, and for the completely random choice game, $\sigma^2=0.25N$. The regular network is a one-dimensional lattice with periodic boundary conditions and coordination number $z=3$ [20], the random network is the ER network of connecting probability 6×10^{-3} [21,22], and the scale-free network is the BA network of $m_0=m=3$ [23]. Therefore, all the networks except the star networks are of average degree $\langle k\rangle=6$. Since the number of edges $\langle k\rangle N/2$ is proportional to the total quantity of information available to agents, the networks used for simulating (except star networks) have the same capacity of information. In the star network, it is not unexpected that the system profit will be reduced when the interaction strength increases. More interesting, in each of

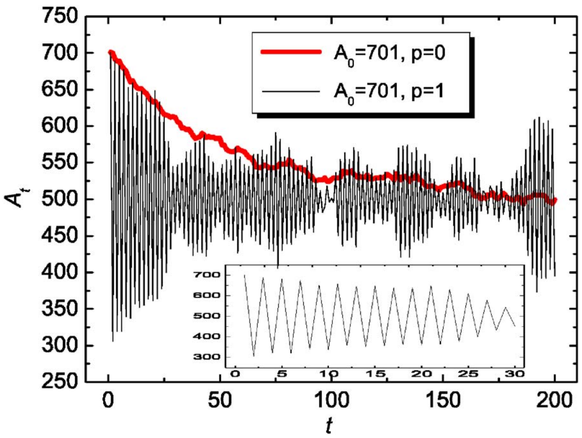

FIG. 2. (Color online) The number of agents choosing +1 vs time. The simulation takes place on regular networks of size $N=1001$. At the beginning, a large event with 701 agents choosing +1 happens. The thick (red) and thin (black) curves show the variety of A_t after this large event for the two extreme cases $p=0$ and $p=1$, respectively. Clearly, in the case $p=0$, A_t slowly reverts to the equilibrium position $A \approx N/2$; while in the case $p=1$, the system displays obvious oscillation behavior. The inset exhibits the oscillation of A_t in the case $p=1$ for the first 30 time steps.

the latter three cases, the system performs better than the random choice game when p is in a certain interval, indicating the self-organized process has taken place within those networks.

Although having the same capacity of information, the dynamic of scale-free networks is obviously distinguishable from that in regular and random networks, indicating that the topology affects the dynamic behavior a lot. Note that, although the topologies of regular and random networks are obviously different—they have completely different average distance and clustering coefficient and so on [24]—the dynamic behaviors are almost the same in those two networks. The common ground is they have almost the same degree variance $\langle k^2 \rangle$. According to the inequality

$$\langle k^2 \rangle_{\text{star}} > \langle k^2 \rangle_{\text{scale-free}} > \langle k^2 \rangle_{\text{random}} > \langle k^2 \rangle_{\text{regular}}$$

and the simulation results, we suspect that a larger degree variance, i.e., greater information heterogeneity, will lead to less system profit.

In Fig. 1, one can see clearly that for all the four cases, the variance σ^2 is remarkably greater than the random choice game at large p. Consider the extreme case $p=1$. If the agents choosing +1 and −1 are equally mixed up in the networks, and the number of agents choosing +1 at present time is A_t, then in the next time step, the expectation of A_{t+1} is $\langle A_{t+1} \rangle = N - A_t$, with departure $|\langle A_{t+1} \rangle - N/2| = |A_t - N/2|$. If at present time A_t is larger than $N/2$, then A_{t+1} will be smaller than $N/2$ most probably, and the departure from $N/2$ will not be reduced in average. Therefore, in the case of $p=1$, when the "large event" happens, that is to say, when A_t is much larger or much smaller than $N/2$ at some time t, there will be a long duration of oscillation after t, in which A skips between up-side $A > N/2$ and down-side $A < N/2$. The oscillation behavior of A is shown in Fig. 2. At the beginning, a large event with $A_0=701$ is given, then the large oscillation goes on about 30 time steps. In $p=1$ case, if A moves apart from $N/2$, the influence (large oscillation behavior) will stand for long time, leading to very large σ^2. However, in the random choice game, whatever A_{t-1}, the expectation of A_t is always $\langle A_t \rangle = N/2$, and the distribution of $A_t - N/2$ obeys a Gaussian form. That is why systems have poor performance at large p compared with the random choice game.

In another extreme case $p=0$, the expectation of A_{t+1} is $\langle A_{t+1} \rangle = A_t(1-m) + m(1-A_t) = A_t + m(1-2A_t)$. Assume $A_t > N/2$, for $0 < m < \frac{1}{2}$, we have $A_t > A_{t+1} > N/2$. Therefore in this case, when m close to zero, no oscillation of A will occur, but A slowly reverts to the equilibrium position $A \approx N/2$ after a large event. One can easily prove that even for very small m, if the iteration time T is sufficiently long, the system profit will be equal to random choice game, which means $\sigma^2 = 0.25N$. This is strongly supported by the simulation results shown in Fig. 1. The thick (red) curve in Fig. 2 is an example for the case $p=0$. At time $t=0$, a large event $A_0=701$ occurs, and then the curve A_t slowly reverts to $N/2$. After about 170 time steps, it arrives at the equilibrium position $A \approx N/2$.

The two extreme cases also exhibit a clear pictures as to why the system profit can be maximized at a special value of p. The interaction mechanism (with probability p) will bring oscillation, while the independent mechanism (with probability $1-p$) will lead to a long reversion process. The former mechanism makes A skip from one side to another, while the latter one keeps A's side. Thus at a proper value of p, the system can quickly arrive at the equilibrium position $A \approx N/2$ after a large event occurs, which leads to more system profit. The existence of an optimal p has been demonstrated in Fig. 1.

Let us now focus on the scale-free case since it may be closer to reality. Firstly, we assume the agents choosing +1 and −1 are equally mixed up in the network. Since there is also no degree-degree correlation for BA networks [25], for arbitrary agent of degree k (here we do not differentiate between node and the corresponding agent), the probability at which he will choose +1 at time step $t+1$ is

$$\eta_1(k,t+1) = p\left(\sum_{i=0}^{k} \frac{i}{k} C_k^i \rho_1^i(t)(1-\rho_1(t))^{k-i}\right) + (1-p)(1-\rho_1(t)) = 1 + 2p\rho_1(t) - p - \rho_1(t),$$

where $\rho_1(t)$ denotes the density of agents choosing +1 at time step t, and $C_k^i = k!/(k-i)!i!$. Since the probability $\eta_1(k,t+1)$ is independent of k, there must be no correlation between agent's degree and profit. In Fig. 3, we report the agent's winning rate versus degree, where the winning rate is denoted by the average score $\langle s \rangle$ for individual during one time step. $p=0.0$ and $p=1.0$ correspond to the completely independent and dependent cases, respectively; $p=0.03$ is the point where the system performs best, and $p=0.4$ is another point where the system profit is equal to the random choice game. One can see clearly that there exists a positive correlation between the agent's profit and degree in the cases p

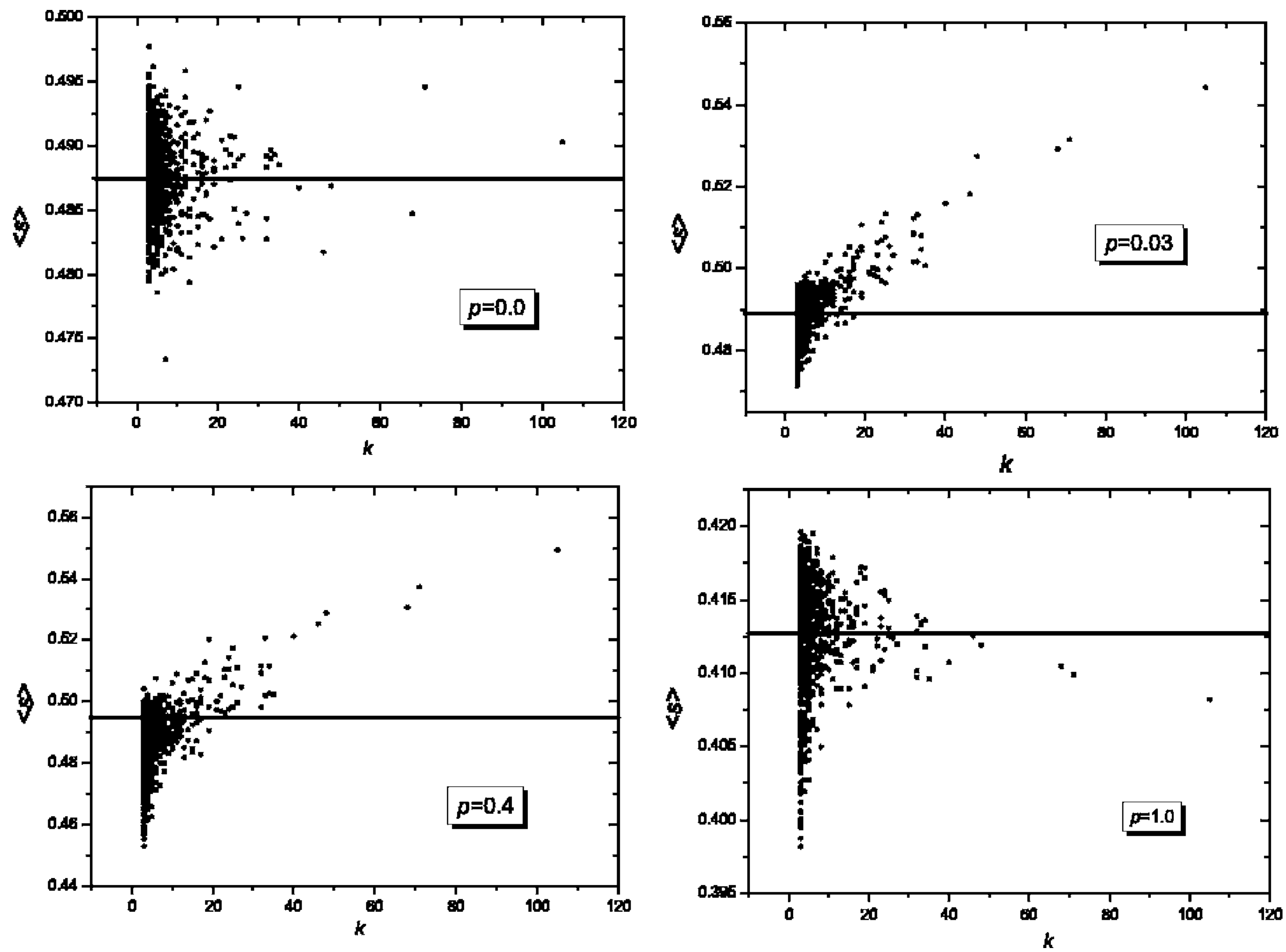

FIG. 3. The agent's winning rate vs degree. Each point denotes one agent and the solid line represents the average winning rate over all the agents. In the cases of $p=0.0$ and $p=1.0$, no correlation is detected. In the cases of $p=0.03$ and $p=0.4$, the positive correlation between agent's profit and degree is observed.

$=0.03$ and $p=0.4$, which means the agents of larger degree will perform better than those of less degree. Figure 4 shows the agent's winning rate as a function of p for different k. It is clear that for a wide extent of p, the agents having more information will gain more. Therefore, the assumption is not true, thus there must be some kind of correlation, which is another evidence of the existence of a self-organized process.

A natural question is addressed: why the agents of large degree will gain more than those of less degree? The reason is the choice of a few hub nodes (i.e., the nodes of very large degree) can strongly influence many other small nodes' (i.e., the nodes of very small degree) choices in the next time step, and those hub nodes can profit from this influence. Denote H the set of those hub nodes and $G_0(t)$ the number of hub nodes choosing +1 at time t. We assume at a certain time step t, $G_0(t)>|H|/2$, which means the number of hub nodes choosing +1 is more than half. This departure will make some nodes connected to those hub nodes, especially the small nodes, choose −1 in time $t+1$ with a greater probability. Because the majority of these small nodes' hub neighbors choose +1 at present, this influence is remarkable and cannot be neglected since the small nodes have only a few neighbors. The more departure $|G_0-|H|/2|$ will lead to the greater influence.

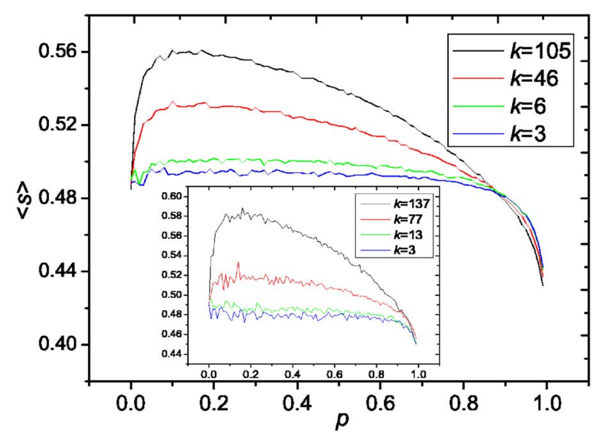

FIG. 4. (Color online) The agent's winning rate as a function of interaction strength. The main plot is obtained by the simulation upon a BA network of size $N=1001$, in which the black, red, green and blue curves from top to bottom represent the four agents of degree 105, 46, 6, and 3, respectively. The inset shows the case upon a BA network of size $N=2001$, where the black, red, green, and blue curves from top to bottom represent the four agents of degree 137, 77, 13, and 3, respectively. It is observed that the agents having more information gain more than those with less information.

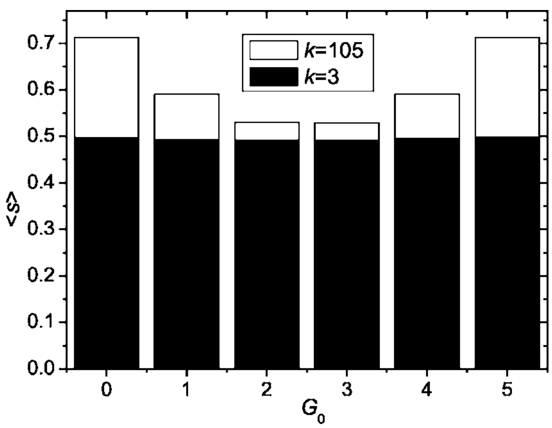

FIG. 5. The agents' winning rate $\langle s \rangle$ under different choice patterns G_0 of the five hub nodes. The simulation takes place on the BA networks of size $N=1001$, and the interaction strength is fixed as $p=0.1$. The hollow and solid histograms represent the winning rates of a hub node ($k=105$) and a small node ($k=3$), respectively. One can see clearly that the winning rates of the small node under different patterns are almost the same as $\langle s \rangle_{k=3} \approx 0.49$, which is obviously smaller than those of the hub node, especially in the cases $G_0=0$ and $G_0=5$.

Figure 5 exhibits an example on BA networks of size $N=1001$, where H contains only five hub nodes of the highest degree. In each time step, all the choices of these five nodes form a choice configuration. There are in total $2^5=32$ different configurations, which are classified into six patterns by identifying the number of agents choosing +1. For example, $G_0=2$ denotes the pattern in which there are two agents choosing +1 and other three choosing −1. Under each choice pattern $0 \leqslant G_0 \leqslant 5$, since $|G_0 - |H|/2| = |G_0 - 2.5|$ is bigger than zero at all times, the hub node can always gain more than the small node. Clearly, under the choice pattern with a larger departure, such as $G_0=0$ or $G_0=5$, the difference of winning rates between the hub node and the small node under these patterns is much greater than the case of a smaller departure.

IV. CONCLUSION

In summary, inspired by the minority game, we propose a model of the Boolean game that can also be considered as a parsimonious model of the local minority game [26,27]. The simulation results upon various networks are shown, which indicate that the dynamic of system is sensitive to the topology of the network, whereby the network of larger degree variance; i.e., the system of greater information heterogeneity, leads to less system profit. The system can perform better than the random choice game. This is reasonable evidence of the existence of a self-organized process taking place within the networks, although only local information is available to agents. We also have found that in heterogeneous networks, the agents with more information gain more than those with less information for a wide extent of interaction strength p. In addition, it is clear that the trends of varying system profit and individual profit are different as interaction strength increases; for example, in the scale-free network with $p=0.5$, the system profit is less than random choice game but the profit of an agent of large degree is much more than that in random choice game.

Although this model is rough, it offers a simple and intuitive paradigm of many-body systems that can self-organize even when only local information is available. Since the self-organized process is considered as one of the key ingredients of the origins of complexity, it might contribute to the understanding of the underlying mechanism of the complex systems.

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